A METHOD OF DETERMINING THE THERMAL CONDUCTIVITY OF METALS AT HIGH TEMPERATURES

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The authors examine the thermal equilibrium of a wire heated electrically in a vacuum with the end kept at constant temperature. A relationship between the overall length of the wire and the electrical resistance of its center section is derived, taking into account heat flow to the ends.

If a wire of length $L = 2l$ is heated in a vacuum by a current I, the equation of thermal equilibrium, taking conduction into account, may be written as

$$\frac{d^2T}{dx^2} - \frac{p\varepsilon\sigma}{kS}(T^4 - T_0^4) + \frac{I^2\rho}{kS^2} = 0. \tag{1}$$

If the temperature of the ends of the wire is fixed and equal to room temperature, then a certain temperature distribution $T = T(x)$, given by (1), will be established along the wire. This equation, together with the boundary conditions was examined in [1].

$$\left.\frac{dT}{dx}\right|_{x=l} = 0; \; T\Big|_{x=0} = T_0 \tag{2}$$

In [1] the authors divided the specimen into sections, in each of which the solution of the equation can be represented as the sum of a power series in the chosen variable. To do this they used the relation

$$\frac{p\varepsilon\sigma}{kS}(T_m^4 - T_0^4) = \frac{I^2\rho}{kS^2}, \tag{3}$$

assuming that there was no outflow of heat due to conduction (specimen infinitely long).

They employed a solution of (1) with boundary conditions (2), expressed in the form

$$2xj\left(\frac{\rho}{k}\,\frac{T_m^3}{T_m^4 - T_0^4}\right)^{1/2} = \sqrt{10}\int\limits_0^{\frac{T-T_0}{T_m}} \{[1-(t+\alpha)]^5 - (1-\alpha)^5 + 5t\}^{-\frac{1}{2}}\,dt. \tag{4}$$

Assuming that in (4) $x = l$ and $T = T_l$, we obtain an equation connecting L and α.

$$Lj\left(\frac{\rho}{k}\,\frac{T_m^3}{T_m^4 - T_0^4}\right)^{1/2} = \sqrt{10}\int\limits_0^{\eta-\alpha} \{[1-(t+\alpha)]^5 - (1-\alpha)^5 + 5t\}^{-1/2}\,dt, \tag{5}$$

where $\alpha = \dfrac{T_m - T_l}{T_m}$, $\eta = 1 - \dfrac{T_0}{T_m}$.

For $\alpha = 0$ the integral diverges at the lower limit. It is easy to show that the divergence is logarithmic in nature. When $\alpha \ll 1$, by suitably dividing up the limits of integration, (5) may be written in the form:

$$Lj\left(\frac{\rho}{k}\,\frac{T_m^3}{T_m^4 - T_0^4}\right)^{1/2} = -\ln\alpha + \Phi(\alpha, \eta). \tag{6}$$

Now the relation between Φ and α does not contain singularities in the vicinity of zero, but

$$\lim_{\alpha\to 0}\Phi(\alpha, \eta) = D(\eta).$$

The function $D(\eta)$ may be written as the sum of a series [1]

$$D(\eta) = \ln \eta + \frac{1}{2}\eta + \frac{1}{16}\eta^2 - \frac{1}{240}\eta^3 + \ldots$$

It may be seen from the foregoing that the asymptotic behavior of the function $L = L(\alpha)$ for $\alpha \to 0$, i.e., when $T_l \to T_m$, may be expressed as follows:

$$L = \frac{1}{j}\left(\frac{k(T_m^4 - T_0^4)}{\rho T_m^3}\right)^{1/2}(-\ln\alpha + D). \tag{7}$$

The quantity

$$\operatorname{tg}\varphi = \frac{1}{j}\left(\frac{k(T_m^4 - T_0^4)}{\rho T_m^3}\right)^{1/2} \tag{8}$$

should be considered as the slope of the asymptote to the graph of the relation $L = L(-\ln\alpha)$, from which it follows that, having the experimental curve of this relation, we can also determine the thermal conductivity k.

If we take into account the temperature dependence of the quantities ρ and ε in the form of linear terms, and assume that the temperature coefficient of the thermal conductivity is substantially less than the temperature coefficients of the resistance β and the total emissivity $\varkappa$, then (1) retains its form, but the coefficients depend on the temperature

$$\rho = \rho_m[1 - \beta(T_m - T)] \text{ and } \varepsilon = \varepsilon_m[1 - \varkappa(T_m - T)].$$

Then relations (7) and (8) may be written as

$$L = \frac{1}{j}\left(\frac{k(T_m^4 - T_0^4)}{\rho_m T_m^3(1+\delta)}\right)^{1/2}(-\ln\alpha + D_1);\ D_1 \neq D \tag{9}$$

and

$$\operatorname{tg}\varphi = \frac{1}{j}\left(\frac{k(T_m^4 - T_0^4)}{\rho_m T_m^3(1+\delta)}\right)^{1/2} = \frac{1}{j}\times$$

$$\times\left\{k\left[\rho_m\left(\frac{T_m^3}{T_m^4 - T_0^4} + \frac{\varkappa - \beta}{4}\right)\right]^{-1}\right\}^{1/2}. \tag{10}$$

Here $\delta = \dfrac{T_m^4 - T_0^4}{4T_m^3}(\varkappa - \beta)$.

It may be concluded from these formulas, that taking into account the temperature dependences of ρ and ε introduces a correction into the relation linking the experimental value $\tan\varphi$ and the unknown value of the thermal conductivity.

To construct the experimental curve $L = L(-\ln\alpha)$, a knowledge of the quantity $\alpha = (T_m - T_l)/T_m$ for various L is required. It can be shown that α may be expressed correct to a fixed factor in terms of the measured resistances of the center section of the specimen.

Let us examine a certain section of the wire of length x_0, which is nonuniformly heated to the temperature $T = T(x)$. Let the temperature range over the whole section be small enough for it to be assumed, with a high degree of accuracy, that the resistivity ρ in this temperature interval is a linear function of temperature with a coefficient β. Let our temperature interval adjoin the temperature T_m. The resistance of the section may be written as:

$$R = R_m[1 - \beta(T_m - T_{\text{eff}})]. \tag{11}$$

Then, because of the linear temperature behavior of the resistivity,

$$T_{\text{eff}} = \frac{1}{x_0}\int_0^{x_0} T dx = T_{\text{av}}. \quad (12)$$

A simple calculation, based on the solution of linear equation (1), shows that the temperature behavior near the middle of the specimen is given by

$$T = T_m (1 - \alpha \operatorname{ch} z) \quad (13)$$

(the point z = 0 marks the middle of the specimen, z being the dimensionless length). Substituting (13) in (12), we find

$$\frac{T_m - T_{\text{eff}}}{T_m} = \alpha \frac{\operatorname{sh} z_0}{z_0}. \quad (14)$$

According to (11), however,

$$\frac{T_m - T_{\text{eff}}}{T_m} = \frac{R_m - R}{R_m} \frac{1}{\beta T_m}. \quad (15)$$

Comparing (15) and (14), we see that α differs from $(R_m - R)/R_m$ by a factor which remains unchanged under the experimental conditions. This means that, due to the logarithm, (7) and (9) retain their form with $\alpha = (T_m - T_l)/T_m$ replaced by $(R_m - R)/R_m$.

The slope remains unchanged, and consequently the working formula retains its previous form.

An expression for the thermal conductivity is obtained from (10):

$$k = j^2 \frac{\rho_m T_m^3 (1+\delta)}{T_m^4 - T_0^4} \operatorname{tg}^2 \varphi = \frac{I^2 R_m}{\Lambda_m S} \left(\frac{T_m^3}{T_m^4 - T_0^4} + \frac{\varkappa - \beta}{4} \right) \operatorname{tg}^2 \varphi, \quad (16)$$

where $\tan \varphi$ is the slope of the asymptote to the curve $L = L\left(-\ln \frac{R_m - R}{R_m}\right)$.

To calculate k from this formula, we require additional information about the temperature behavior of the electrical resistance and a knowledge of the average coefficient of linear expansion in the range of temperatures studied. Determination of the respective coefficients does not present great difficulty from the experimental point of view. Determination of the coefficient x itself does not, in general, require the setting up of additional experiments, since ε is measured for each temperature T_m in accordance with the proposed method using (3).

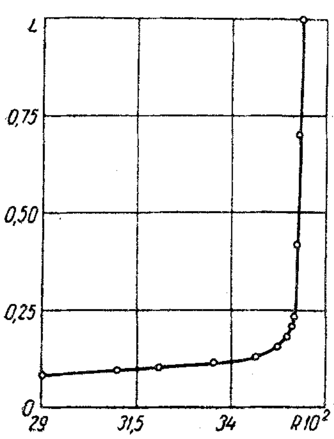

Fig. 1. Total length of wire L as a function of the resistance R of its center section for the same current.

The specimen to be examined was placed inside an evacuated glass vessel and soldered to solid copper contacts to achieve constant temperature at the ends. A platinum wire 0.3 mm in diameter was used. Two platinum wires of 0.05 mm in diameter were welded to the center section of the specimen to serve as potential leads. The resistance was measured by the usual potentiometric method. The initial length of the wire was so chosen that the specimen could be considered infinitely long, in the sense that heat flow from the center section to the ends due to thermal conduction was practically zero. The ratio of length to diameter was approximately 10^3. Accordingly, measurement of the resistance of the center section of a wire of this length for a given heater current yielded a value of R_m corresponding to heating of the section to the temperature T_m. By shortening the wire symmetrically from both ends, and each time measuring at the same current, we obtained a set of values of the length L and corresponding values of the resistance of the center section R (see Fig. 1).

Calculations have shown that the relative error in the measured resistance introduced by the presence of the potential leads, insofar as these distort the temperature field of the specimen, is approximately 0.3%, while the overall error of the method does not exceed 5%.

Fig. 2 gives a graph of L as a function of $-\ln((R_m - R)/R_m)$ (for T_m = 978°K). The graph shows that the experimental curve does not have the form of a curve approaching its asymptote without limit. The divergence of the curve obtained from its asymptote in the region of large arguments is due to error in the measuring equipment; for very small differences $R_m - R$, the relative error $\Delta(R_m - R)/(R_m - R)$ ceases to be small, as a result of which the quantity under the log sign has the average error of the equipment as its limit, which also limits further increase in the absolute value of the logarithm itself. This is expressed in that the experimental curve enters, as it were, a "saturation" region, as may be seen in Fig. 2. The asymptotic formula (9) is realized experimentally to a satisfactory degree over a certain range of lengths of our specimen (the linear part of Fig. 2). This range is characteristic of the given specimen and the given measuring equipment. To evaluate the thermal conductivity we used the slope of the straight portion of the experimental curve. The difference between the slope of the straight section and the true slope of the asymptote represents the basic error of the method. The error was calculated with allowance for this difference.

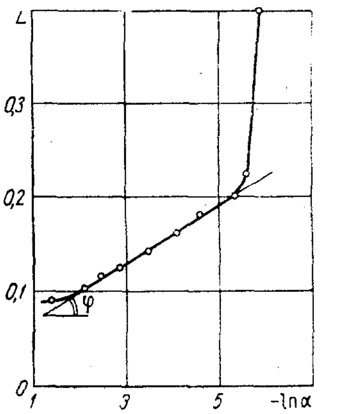

Fig. 2. Total wire length L vs. $-\ln\alpha$, $\alpha = (T_m - T_l)/T_m$

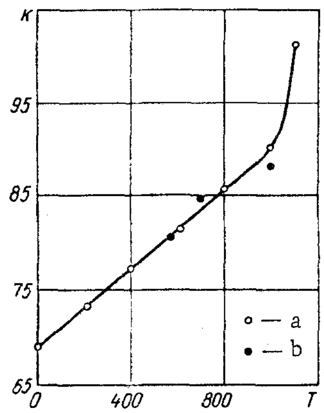

Fig. 3. Comparison of the authors' values for the resistance of platinum (a) with handbook data (b)

To check the method, we measured the thermal conductivity of platinum at various temperatures. The corresponding results are given in Fig. 3, and, within the limits of experimental error, are in satisfactory agreement with results of other authors [2].

The proposed method is suitable for use on wire specimens at temperatures ranging from a little above room temperature right up to the region of the melting point.

NOTATION

T – absolute temperature; I – current intensity; j – current density; L, l, Λ, x – lengths; S – area; p – perimeter; R – electrical resistance of center portion of specimen; ρ – resistivity; k – thermal conductivity; σ – Stefan-Boltzmann constant; ε – (dimensionless) emissivity; β – temperature coefficient of ρ; ϰ – temperature coefficient of ε; α, δ, η – dimensionless temperature differences; z – dimensionless length; R_m – resistance section heated uniformly to temperature T_m; T_{eff} – effective temperature, corresponding to a given temperature distribution in the section, and a given temperature coefficient β. Subscripts: m – absence of heat flow to the ends of the specimen; R_m and Λ_m – respectively, resistance and length of center section of wire at temperature T_m; T_l – temperature of middle of specimen with heat flow to the ends; T_0 – room temperature; x_0, z_0 – fixed length.

REFERENCES

1. S. C. Jain and K. S. Krishnan, Proc. Roy. Soc., A 222, N 1149, 1954.
2. N. B. Vargaftik, Thermophysical Properties [in Russian], Gosenergoizdat, 1956.

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